3 Mathematical Modeling for Decision Making

3.2. Multiscale coherence in the analysis of gamma rays in well characterization

Multiescale coherence in the analysis of gamma rays in well characterization


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Abstract. We perform a multiscale wavelet analysis to gamma rays petrophysical registers of wells to identify electro-facies and electro-facies associations at different scales comparing the results with those obtained through standard geological procedures on the same registers. We then apply coherence analysis, a type of covariance cross-correlation based on the wavelet power spectra of gamma ray series of different but related wells. We show that coherence is a valuable technique to identify common features in well registers and also we show that they provide a feasible technique to associate or discriminate between wells belonging to the same or different sandstone formations. All our analysis are based on oil field pilot and our conclusions relate to actual cases in Chicontepec Formation of the Tampico-Misantla basin in Mexico.

Keywords: Wavelet spectrum, wavelet coherence, petrophysical registers, oil wells, Chicontepec Formation.

1 Introduction

To obtain a reliable reservoir characterization, it is important to have a proper identification and mapping of rock facies. This means that reservoir characterization requires the lithological and petrophysical knowledge of the study area. In this sense, the identification of rock types is a main objective in oil exploration to find reservoir and seal rocks (e.g. sandstones) [2]. Facies identification is usually based on the inspection of core data and well logs. Facies are rock classes or sets with well defined characteristics that can be paleontological, (fossils) or lithological (form, size, grain disposition and distribution and mineral composition) that help to understand when and how a given rock was formed [7]. Analysis of facies association can be accomplished through the use of pattern recognition techniques such as wavelets [7]. When the geological information is incomplete or nonexistent, the wavelet analysis can help us to identify the facies association.
Cyclicity detection of sedimentary strata is of importance for the understanding of those factors that control cyclic patterns and changes in characteristics in sedimentary deposition [4]. We performed our test on the field arrangement F\(^1\) located at the Chicontepec Formation interval, using Gamma Ray logs, as the natural radioactivity of rocks [16]. Natural radioactivity of the rock used as a simple clay mineral content indicator could be linked to energy fluctuations of depositional processes which control grain size distributions.

In this work we use the signal of Gamma Ray well logs to recognize association of electro-facies (eF) in the F pilot. The exploratory analysis performed includes 8 wells, seven of them belonging to the well arrangement F; well number eight is not in the arrangement but is the one with cores nearest to the arrangement. The wavelets were applied to Gamma Ray well logs and wavelet coherence analysis was applied to contribute on the correlation of the wells mentioned above.

In many applications, it is desirable to quantify statistical relationships between two non-stationary signals. The coherence is used to determine the association between two signals. It is a direct measure of the correlation between the spectra of two time-series [6]. The coherence quantify the relationships between two non-stationary signals.

In the present exploratory analysis, we used the Wavelet Toolbox in Matlab software version 7.12 to perform the wavelet transform calculation.

2 Methods

The wavelet transform has been used extensively in many areas of science and technology. We give here a very brief description of wavelet and refer the reader to the references list for further information. Here we have followed [1]. A wavelet is a function \( \psi(t) \) such that the following are satisfied

- The quantity \( E \) defined as the energy of the wavelet must be finite, i.e.,
  \[
  E = \int_{-\infty}^{+\infty} |\psi(t)|^2 dt < \infty.
  \]

- If \( \hat{\psi}(f) \) denotes the Fourier transform of \( \psi(t) \) given as
  \[
  \hat{\psi}(f) = \int_{-\infty}^{+\infty} \psi(t)e^{-i2\pi ft} dt,
  \]

  then the following must hold
  \[
  C_g = \int_{0}^{\infty} \frac{|\hat{\psi}(f)|^2}{f} df < \infty.
  \]

\(^1\) For reasons of confidentiality the specific name and location of the site is omitted. It is referred to in these notes as field arrangement F.
2.1 The wavelet transform

Wavelet analysis represents an advance from Fourier analysis through a windowing technique with flexible regions. Wavelet transform uses a window function whose radius increases in space (reduces in frequency) while resolving the low-frequency contents of a signal [13].

A last requirement is that, for complex wavelets, their Fourier transform must be real and vanish for negative frequencies. One consequence of the above conditions is that the wavelet has no zero frequency component. Also, the quantity $C_g$ is known as the admissibility constant and depends on the chosen wavelet. The continuous wavelet transform (CWT) of a continuous signal $x(t)$ with respect to the wavelet function is defined as

$$T(a, b) = w(a) \int_{-\infty}^{\infty} x(t) \psi^*\left(\frac{t-b}{a}\right) dt,$$

where $w(a)$ is a weighting function, $\psi^*$ denotes complex conjugation, and $a$ and $b$ are the dilation and translation parameters, respectively. It is frequent to use $w(a) = 1/\sqrt{a}$.

The total energy in a signal is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt.$$

As a generalization of the above, the relative contribution of the signal energy at an specific scale $a$ and location $b$ is defined as

$$E(a, b) = |T(a, b)|^2,$$

which is a two-dimensional energy function. Plotting $E(a, b)$ for different values of $a$ and $b$ produces the scalogram of the signal. For real wavelets the scalogram and the CWT plot since, in this case, one is only the squared magnitude of the other.

The function

$$E(a) = \frac{1}{C_g} \int_{-\infty}^{\infty} |T(a, b)|^2 db$$

is the wavelet energy distribution that allows the identification of dominant energy scales in the signal $x(t)$.

The wavelet transform of a continuous signal $x(t)$ can be represented for discrete $a$ and $b$ parameters. The usual discretization takes the form

$$\psi_{m,n}(t) = \frac{1}{\sqrt{a_0}} \psi\left(\frac{t-nb_0a_0^m}{a_0^n}\right),$$

whence the wavelet transform of $x$ can be represented as an inner product

$$T_{m,n} = < x, \psi_{m,n} >,$$

where these coefficients are known as the wavelet coefficients of the signal.
2.2 The discrete wavelet transform

Usually \( a_0 = 2 \) and \( b_0 = 1 \) are taken, given rise to the so-called dyadic grid arrangement, that renders

\[
\psi_{m,n}(t) = 2^{-m/2}\psi(2^{-m}t - n).
\]

These dyadic grid wavelets are chosen to be orthonormal which implies that the wavelet coefficients are not redundant for the signal reconstruction. The Discrete Wavelet Transform (DWT) \([7]\) is defined then using the dyadic grid wavelets as

\[
T_{m,n} = \int_{-\infty}^{\infty} x(t)\psi_{m,n}(t)dt.
\]  

(2)

2.3 The continuous wavelet transform

The continuous wavelet transform (CWT) \([10]\) of a signal \( x(t) \) is defined as \([17]\):

\[
CWT(a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t)W_{ab}(t)dt = \int_{-\infty}^{\infty} x(t)W\left(\frac{t - b}{a}\right)dt
\]

where \( W \) is a function that is generated from the mother wavelet by translation and scaling; or in terms of spectral representation

\[
CWT(a, b) = \sqrt{|a|} \int_{-\infty}^{\infty} x(\omega)W^*(a\omega)e^{ib\omega}d\omega
\]

where \( \ast \) is the complex conjugate, \( \omega \) frequncy and \( i = \sqrt{-1} \). The signal \( x(t) \) must be of a finite energy. In this case, the signal \( x(t) \) can be reconstructed or synthesized by means of the inverse continuous wavelet transform \([17]\), defined as:

\[
x(t) = C_g \int \int \text{CWT}_x(a,b) \frac{1}{\sqrt{|a|}} W\left(\frac{t - b}{a}\right) dbda
\]

\( C_g \) is a constant depending on the wavelet to be used (admissibility constant).

2.4 Wavelet coherence

In this section we follow \([18]\). Given two time series \( x \) and \( y \) we can compute their CWT as \( T(a, b)^x \) and \( T(a, b)^y \). The cross-wavelet spectrum of the two series is

\[
T(a, b)^{xy} = T(a, b)^x T(a, b)^y^*.
\]

We, therefore, can define the cross-wavelet power as

\[
|T(a, b)^{xy}|.
\]

The statistical significance of the wavelet coherence is determined by simulation following the procedure of Johansson \([12]\) using the same null hypothesis that reads: the variability of the observed time series is equivalent to the expected variability of a random process with similar first order autocorrelation \([12]\).

The analysis reported here is done on petrophysical registers from the F Field and performed using Matlab.
3 Description of the arrangement of wells

The Chicontepec formation is an important target for oil exploration in Mexico. Geographically, the Chicontepec deposits are located in the states of Puebla and Veracruz, and belong to the Tampico-Misantla basin; it is divided into 29 polygons that represent the same number of oil fields. The Chicontepec sedimentary sequences consist of sandstone bodies whose potential for oil production depends on its lateral continuity and petrophysical characteristics [3].

For the analysis described here we consider a well arrangement F with wells denoted as: \(w_1, w_2, w_3, w_4, w_5, w_6\) and \(w_7\). As mentioned before the well \(w_8\) is outside the arrangement but it is considered here because it is the well with cores nearest to the arrangement and serves as a reference for the calibration of our methods. Figure 1 shows the distribution of the wells whose registers are treated here.

![Well arrangement distribution](image)

Fig. 1: Well arrangement distribution. This well arrangement is referred to as F in this work. Distances between wells are 400 meters, \(w_8\) is 1200 meters from other wells.

The analysis for identification of electro-facies by geologists with which we compare our results in this exploratory analysis is as follows: In the pilot arrangement F four electro-facies, denoted by \(eF_i\), \(i = 1, 2, 3, 4\), were proposed by intervals of radioactive signal (clay mineral content) from the gamma ray log, and are described as follows:

- **eF1.** Sandstone (Ss).
- **eF2.** Argillaceous sandstone (Ss-Sh).
- **eF3.** Sandy shale (Sh-Ss).
- **eF4.** Shale (Sh).

These electro-facies were grouped in seven electro-facies associations (eFA) as follows:

- **eFA1** Predominance of eF1.
- **eFA2** Predominance eF1 with less than 10% of eF4.
- **eFA3** Predominance eF1 with 11 to 40% of eF4.
3 Mathematical Modeling for Decision Making

3.1. Lotka-Volterra system applied to the generation of technological innovation


**eFA4** Intercalation of eF1 with eF4 in the same percentage (40 to 60%).

**eFA5** Predominance of eF4 with 11 to 40% of sandy horizons.

**eFA6** Predominance of eF4 with less than 10% of sandy horizons.

**eFA7** Predominance eF4.

In oil and gas industry, depth in a well is the distance between a reference point (kelly bushing), or rotary table elevation, and a given point in the well. The developed meters (MD) measure distance along the path of the borehole, and the true vertical depth (TVD), gives absolute vertical distance between the datum and the point in the wellbore by correcting for deviation path. In perfectly vertical wells, TVD equals MD [14].

4 Wavelet analysis

The exploratory analysis reported here was made to select the wavelet that best identifies the electro-facies by using gamma ray well logs. There are several types of wavelet, for example, Haar wavelet, Morlet wavelet, Meyer wavelet, Mexican hat wavelet, Daubechies wavelet, Symlet wavelet, Coiflet wavelet, Gaussian wavelet, among others [18]. Some wavelets have been developed for specific applications [15]. We have explored several of them to determine the one that best suited to determine electro-facies associations in gamma ray registers. Any discussion of wavelet begins with Haar wavelet, the first and simplest. It was introduced by Alfred Haar in 1910 [11]. Haar wavelet is discontinuous, and resembles a step function (it represents the same wavelet as Daubechies 1). The Haar mother wavelet is

\[
\psi(x) = \begin{cases} 
1 & \text{if } 0 \leq x < \frac{1}{2}, \\
-1 & \text{if } \frac{1}{2} \leq x < 1, \\
0 & \text{another value}
\end{cases}
\]

The Haar wavelet is the one used in this work to illustrate our results. Other wavelets can be applied but those results will be reported elsewhere. Having defined the wavelet best suited to our exploratory analysis the task that we have is to determine the appropriate scale that will enable us to identify the electro-facies associations.

4.1 Scale determination

Attempting to find an appropriate scale for the analysis of gamma ray signals we study the thickness in electro-facies associations. We first illustrate the approach considering the well \(w_8\). For this well it has been determined that the Chicontepec Formation is located between 856.15 and 1299.36 [MD].
Fig. 2: Gamma ray signal and electro-facies associations for well $w_8$.

As reference, Figure 2 shows the gamma ray signal and electro-facies associations determined by geologists visual inspection for well $w_8$, calibrated with core lithofacies analysis. The thickness for each electro-facies association ranges from 6 meters to 68 meters.

These lengths are used as base to define the scales in the wavelet analysis. First consider thicknesses between 2 and 10 meters (corresponding to the electro-facies associations with the smallest length), then we used thicknesses between 46 and 76 meters, which correspond to the largest electro-facies associations.

Finally we consider thicknesses between 10 and 20 meters because in this range scale we found a greater number of electro-facies associations.

Applying the Haar wavelet transform to the gamma ray signal we obtain Figure 3.
In Figure 3 we consider three signals at different scales: A, B and C. A is Haar wavelet transform for the electro-facies associations of the smallest length (2-10 meters). B considers the Haar wavelet transform for the electro-facies associations for lengths in the range 46-79 meters and C shows the Haar wavelet transformed for electro-facies associations of lengths 10-20 meters.

We note that the wavelet identifies the eFA7 (predominance eF4) electro-facies and eFA6 (predominance eF4 with less than 10% of sandy horizons).

Note that the wavelet that identifies shales in all areas is C which corresponds to lengths between 10 to 20 meters. In zone 6 we observe that A and B identifies the electro-facies associations very weak signal. In Zone 5 and Zone 4 A identifies very weak signal and B does not identify at all. In summary A is a weak shale identifier and B does not identify shale at all.
A successful electro-facies associations identification using the Haar wavelet transform depends on the fact that most electro-facies associations have thicknesses between 10 and 20 meters (C). In the wavelet scale this corresponds to wavelengths of 65 to 125.

We offer next an exploratory analysis for the arrangement wells \( w_4 \), \( w_5 \) and \( w_6 \) using the Haar wavelet.

![Wavelet Transform Example](image)

(a) \( w_4 \) well.  
(b) \( w_5 \) well.

Fig. 4: Electro-facies associations obtained by geologist visual inspection and Haar wavelet signal for \( w_4 \) and \( w_5 \) wells.

In Figure 4 shown consist of two panels each, the first one shows the electro-facies associations obtained by traditional methods (geologists visual inspection), and the second the corresponding Haar wavelet transform.

For each well, we excluded from the analysis the well log intervals that does not belong to the Chicontepec formation. Well \( w_4 \) Chicontepec formation ranges from 927.5 to 1329.36 [MD] and well \( w_5 \) from 1059.27 to 1465.12 [MD].
The wavelet analysis in Figure 4 is carried out considering only thicknesses between 10 to 20 meters.

As a final illustration we show in Figure 5 the wavelet coefficients for the gamma ray log of well \( w_8 \). It can be seen that the peaks with the highest coefficients of the curve correspond to the predominance \( eF4 \) (shale electro-facies).

Figure 5 consists of two panels, one at the left shows the continuous wavelet coefficients calculated using the Haar wavelet transform from the original gamma ray log, it is the graph of the average normalized wavelet coefficients, the scale values determine the degree to which the wavelet is compressed or stretched. We use the scale between 10 to 20 meters. Note that the maxima of this plot corresponds exactly to \( eFA5 \) and \( eFA6 \).

Fig. 5: Haar wavelet coefficients and electro-facies associations for well \( w_8 \).
5 Using wavelet coherence to identify patterns in gamma ray registers of different wells

We now proceed to the study of coherence between the wavelet spectra of two different wells using the gamma ray signal. Performing the wavelet coherence between the arrangement of the wells studied \((w_1, w_2, w_3, w_4, w_5 \text{ y } w_6 \text{ and } w_7)\), we conclude that well \(w_8\) cannot be compared to wells from the arrangement because the gamma ray data was taken every 15 centimeters, and all the measurements in the wells from the arrangement were taken every 10 centimeters.

The wavelet coherence analysis is performed for each pair of wells. It is desirable to quantify statistical relationships between two non-stationary signals. The coherence function is a direct measure of the correlation between the spectra of two series [6]. While a continuous wavelet transform is a common tool for analyzing localized intermittent oscillations in a series, it is very often desirable to examine two series together that may be expected to be linked in some way. In particular, to examine whether regions in time frequency space with large common power have a consistent phase relationship suggesting causality between the time series being compared. Many time series are not normally distributed and we suggest methods of applying the continuous wavelet transform to such time series [9].

In this exploratory analysis we use the Morlet wavelet to find the relationship between two depth series by wavelet coherence.

The Morlet wavelet is a particular complex continuous wavelet and is defined as

\[
\psi(t) = \pi^{-1/4} \exp(-i2\omega_0 t) \exp(-t^2/2).
\]

This wavelet is the product of a complex sinusoidal \(\exp(-i2\omega_0 t)\) and a Gaussian envelope \(\exp(-t^2/2)\) where \(\omega_0\) is the central angular frequency of the wavelet. The term \(\pi^{-1/4}\) is a normalization factor to ensure unit variance [5].

Considering the well aerial distribution shown in Figure 1, we use well \(w_4\) as the center of the arrangement and analyze a pairwise wavelet coherence between all the wells around; once this is done we perform the wavelet coherence between wells \(w_1\) and \((w_3, w_4, w_6, w_7)\), \(w_7\) and \((w_3, w_4, w_5, w_6)\).

The problem that one faces here is: given the high heterogeneity of the sandstone bodies on which the well arrangement sits, one has to determine the likely similarity between electro-facies associations that may help in solving sandstone bodies correlation alternatives.

We compare the wavelet spectra of wells \(w_1\) and \(w_3, w_4\) and \(w_5, w_6\) and \(w_7\) in figure 6 and figure 7, the white color indicates areas of high coherence (covariance) between signals whereas lighter tones signify low coherence.
3 Mathematical Modeling for Decision Making

3.1 Lotka-Volterra system applied to the generation of technological innovation

Fig. 6: Wavelet coherence between wells $w_1$ and $w_3$ (A), $w_1$ and $w_4$ (B), $w_1$ and $w_6$ (C), $w_1$ and $w_7$ (D). The Zone 1 shows the presence of patches, which identify equivalent sandstone intervals.
Fig. 7: Wavelet coherence to wells $w_7$ and $w_3$ (A), $w_7$ and $w_4$ (B), $w_7$ and $w_5$ (C), $w_7$ and $w_6$ (D). The Zone 1 and Zone 2 and Zone 3 shows the presence of patches, which identify equivalent sandstone intervals.
Blue indicates areas of no coherence. At very short wavelengths, coherence is almost zero, indicating the random nature of the compared signals, imposed by the scale resolution of the gamma ray tool (10 cm). As the wavelength increases more clear patterns appear. It is apparent that coherence arises in patches defined by boundaries of very low coherence. Note that the low coherence boundaries (which are almost lines) form a sort of tree which branches more and more as the wavelength decreases.

Also note that at wavelengths of 365 and 469 the boundary lines of low coherence spread to form small patches (bird-like blue patches), ca. between $w_1$ and $w_7$, $w_1$ and $w_6$, $w_1$ and $w_3$, $w_1$ and $w_7$. This evidence coincides with seismic analysis (in curse) which apport elements to seek correlation of sandstone intervals.

**Conclusions**

Wavelets are powerful analytic tools to identify patterns present on signals. They have been amply used in petrophysical register analysis. We here show the first steps in develop a methodology to incorporate wavelet analysis combined with coherence of wavelet spectra to add in efforts to perform lithofacies correlations among wells with frequent lithofacies changes. Our methodology has to be used in conjunction with seismic interpretation and geological analyses (core well description, oil-field analogue characterization, etc.) since this approach is by nature local at the level of wells scale. Elsewhere we will publish the more detailed studies that incorporate all of these components of characterization applied to concrete examples in the Chicontepec oil-fields.

Acknowledgements: The work described in this paper was supported by Mexican Petroleum Institute (IMP) and Y.00114. Senner Conacyt. New methodologies and tools considering static and dynamic characterization of the fractal properties of the oil fields project. We gratefully acknowledge the assistance of Ing. Honorio Ramírez Jiménez and Ing. Judith Callejas Moreno and Ing. Alejandro Lopez Zuñiga in the compilation of this paper.

**References**